













nice classroom activity which get at the notion of fraction as *operator* is the “Estimation of Fractions” activity from the *Mathematics Curriculum and Teaching Program* materials, which involves initially finding  $\frac{2}{5}$  of the way across the blackboard (Lovitt & Clarke, 1988).

### Taking the activity a little further

In debriefing the chocolate game, I also encourage students to think about their answers to the following questions:

- (directed to the individuals at the chairs) If, at the end, you had the choice to move to a different chair, would you do so?
- (directed to the students watching the task) Where would you *choose* to stand in the queue? Is it best to go first or last?
- What strategies would you use if you were in the line?

I then either act out or discuss the following two situations:

1. The case where the chocolate is out of the packet and clearly already subdivided. In this discrete case, the students are now using the fraction as an *operator* notion, as they calculate, say,  $\frac{1}{3}$  of 24 blocks or  $\frac{1}{5}$  of 20 blocks.
2. The case where there are more blocks of chocolate at a chair than people. In this case, a context is available to discuss improper fractions, where the mixed number equivalent is either obvious or can be easily determined. For example, five blocks shared between three is  $\frac{5}{3}$ , but by breaking each block into three, we can see that each person would get five thirds, which could form one and two-thirds blocks.

No doubt, the reader may have thought of other directions in which this activity might proceed, but even to this point, I would claim that this one lesson has enabled the emergence of a range of important ideas, and broadened hopefully the notion of what fractions are all about for many students.

### Another problem worth posing to middle school students

A task that forms part of the interview we are developing, originally taken from Lamon (1999), which usually brings out a range of interesting responses, is the following (Figure 3.4):

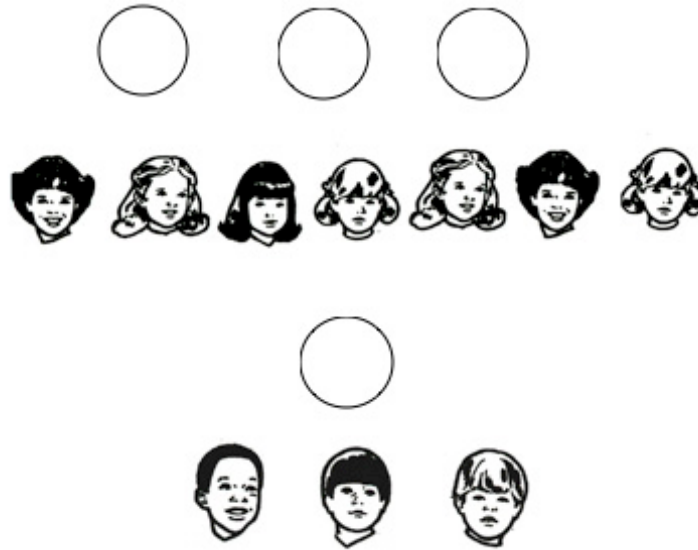


Figure 3.4

We explain that three pizzas are shared evenly between seven girls, while one pizza is shared evenly between three boys. The question is posed, “Who gets more pizza: a boy or a girl?”

Students who understand the notion of fraction as division in a way that they can use it, will quickly conclude that each girl gets  $\frac{3}{7}$  of a pizza and each boy gets  $\frac{1}{3}$  of a pizza, and then think about which of these two fractions is larger. If they “benchmark” to one-half, they will usually conclude that  $\frac{3}{7}$  is larger because it is almost one-half. Not surprisingly, in light of the earlier discussion, few students or teachers use this strategy, unless they have recently played the chocolate game!

Other more interesting methods often arise. For example, we are always pleased when a student will give one of the following two responses:

- “Well, three boys share one pizza, so I’ll give the first three girls the first pizza, and the next three girls the next pizza, leaving a whole pizza for the seventh girl, which must mean [on average] that the girls get more.”
- “Seeing as the boys get one-third each, I imagine dividing the girls’ pizzas into thirds, which would give a total of nine thirds. As there are seven girls, there will be two thirds left over, so [on average] the girls get more.”

Taking the activity a little further, students can be challenged to follow up their answer to “Who gets more?” with a quantification of how much a girl and boy get, respectively. Further, how much more does one get than the other?

In an easier variation of this task, we asked 323 Grade 6 students to indicate how much each person would get if three pizzas were shared



between five people (Clarke, Roche, Mitchell, & Sukenik, 2006). Only 30% of students at the end of the Grade 6 year could give a correct answer to this question: 12% did so mentally, while 18% used a drawing to reach their answer.

The reader is invited to try any of the problems given in this article with individuals, small groups or the whole class, and explore their potential for assessing and developing student understanding, leading to a broader, more connected and applicable notion of fractions. As Kilpatrick, Swafford, and Findell, (2001) note, “sharing can play the role for rational numbers that counting does for whole numbers” (p. 232).

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