

Investigating the maths inside:

Big data, better hospitals

Activity 4

Waiting, waiting



Standing in a queue can be very frustrating. Queuing for service at a hospital can have much more serious consequences.

What factors make queues form? How can queues be managed?

# Introduction

CSIRO has developed the Patient Admission Prediction Tool (PAPT) which enables hospitals to make significant improvements in efficiency, reduce waiting times, and provide timely access to emergency care.

*The data used in this activity have been provided by Queensland Health’s Healthcare Improvement Unit and are used with their approval.*

# Simulating a queue in the classroom

Your teacher will select four students:

* a server, to stand behind a desk/table ready to receive customers
* a timer, to indicate when 10 seconds has elapsed for each customer
* a roller, to roll a six-sided die every 20 seconds to indicate how many new customers are added to the queue
* a recorder, to record the time on a whiteboard that each customer stays in the queue.

The rest of the class will be the customers and are arranged in a line.

The activity starts when the die is rolled.

The number on the die indicates the number of customers arriving at one time (1, 2, 3, 4 or 5, with the 6 counting as a zero). Roll once to find out how many customers are waiting in the queue when the server begins serving. The server spends 10 seconds serving each customer. The timer needs to indicate when the 10 seconds is up, perhaps by ringing a bell.

The die is rolled every 20 seconds, the number on the die indicating the number of customers added to the queue.

Each customer notes the length of time that they waited (e.g. 0, 10, 20, 30 seconds) and relates the time to the recorder to scribe onto the whiteboard.

## Observation

Does a queue form? When? What is the shortest waiting time? The longest? How long is the longest queue? Are there any times when customers do not wait?

## Changing the situation

How do you think you could stop a queue from forming? Try it. What happened?

# Exploring simple queues

Markets often have food stalls and queues can develop as customers wait to be served. Consider a food stall where it takes each customer one minute to be served. What would happen if two people arrived at the stall every minute?

A fixed number of customers arriving is not very realistic. A more likely situation is that the number of customers arriving varies. Consider the situation where either 0, 1, 2 or 3 customers arrive every minute.

What is the mean number of customers arriving every minute?

What is likely to happen in this situation?

We can look in more detail by drawing up a table to follow what is happening.

|  |  |  |  |
| --- | --- | --- | --- |
| Minute | Number of customers that arrive | Number of customers served | Number of customers in queue |
| 1 | 1 | 1 | 1 – 1 = 0 |
| 2 | 0 | 1 | 0 + 0 – 1 = -1 |
| 3 | 0 | 1 | 0 + 0 – 1 = -1 |
| 4 | 2 | 1 | 2 + 0 – 1 = 1 |
| 5 | 2 | 1 | 2 + 1 – 1 = 2 |

There are two negative numbers in the ‘Number of customers in queue’ column. What does this mean?

Your teacher will supply you with a table where the ‘Number of customers that arrive’ is randomly between 0 and 3, and where each customer can be served in one minute. Complete the table, which considers the first ten minutes of serving.

Your teacher will provide you with a blank table. Complete this table with a different set of numbers in the second column, ‘Number of customers that arrive’, generated randomly by using a spinner with the numbers 0, 1, 2 and 3. Compare the results for the final number of customers in the queue with others. Are they the same?

What do you think will happen in the first hour? The first two hours? Beyond two hours?

# Exploring queues using spreadsheets

Using an Excel spreadsheet, we can randomly allocate the number of customers arriving and investigate a variety of different situations.

Your teacher will provide you with a spreadsheet to investigate different queueing situations over a period of 30 minutes. Note that in the provided spreadsheet negative numbers appear as 0.

To simulate the forming of a number of different queues, refresh the spreadsheet ten times and look carefully at the lengths of the queues formed.

Are there situations where the customers are waiting for a long period of time? If the owner wants to keep the customers happy, what should they do?

Should another server be considered?

Can you explain what the formulas in cells D6 and F6 of the spreadsheet are doing?

## Increasing the number of servers

Change the spreadsheet to indicate that with two servers, two customers are served every minute. Refresh the spreadsheet ten times.

Does this solve the problem of customers waiting too long? Are more servers required? Are the servers idle for any period of time? Is this significant when considering the profitability of the stall?

What happens when the number of servers is increased? Is there an ideal solution which would ensure limited waiting time but still enable the owner to make a profit?

If the market becomes more popular, the number of customers could be 0, 1, 2, 3, 4, or 5, each of which is equally likely. How many servers would be required? What would the owner need to consider to continue to make a profit?

# Using hospital data

Your teacher will provide you with a spreadsheet of hospital admissions for every day in July. Assume each patient will take 30 minutes to be treated. A doctor can therefore see two patients each hour.

Each student in the class will be given the data from a different day to investigate. Set up your own spreadsheet for a 24-hour period for the day you are given, assuming two patients can be seen each hour.

The table below shows what happens on Day 12, the day that had the highest daily admissions.

|  |  |  |  |
| --- | --- | --- | --- |
| Hour | Number of patients | Patients seen | Patients waiting |
| 0 | 0 | 0 | 0 |
| 1 | 2 | 2 | 0 |
| 2 | 3 | 2 | 1 |
| 3 | 0 | 2 | 0 |
| 4 | 3 | 2 | 1 |
| 5 | 3 | 2 | 2 |
| 6 | 2 | 2 | 2 |
| 7 | 3 | 2 | 3 |
| 8 | 2 | 2 | 3 |
| 9 | 3 | 2 | 4 |
| 10 | 0 | 2 | 2 |
| 11 | 2 | 2 | 2 |
| 12 | 0 | 2 | 0 |
| 13 | 0 | 2 | 0 |
| 14 | 5 | 2 | 3 |
| 15 | 2 | 2 | 3 |
| 16 | 1 | 2 | 2 |
| 17 | 3 | 2 | 3 |
| 18 | 3 | 2 | 4 |
| 19 | 6 | 2 | 8 |
| 20 | 4 | 2 | 10 |
| 21 | 4 | 2 | 12 |
| 22 | 2 | 2 | 12 |
| 23 | 5 | 2 | 15 |
| 24 | 2 | 2 | 15 |
| Totals | **60** |  |  |

In this situation, there appears to be a problem with the numbers in the queue building to unacceptable levels. How does it compare with the day you investigated and the days investigated by other students? Is there an optimum number of patients that can be seen by one doctor in a reasonable time?

To help answers these questions draw a line graph in Excel of the data from the day you were given with ‘patients waiting’ on the vertical axis and ‘hour’ on the horizontal axis.

The hospital could employ a second doctor to reduce the waiting times. Investigate the best way to use a second doctor efficiently.

Share your results with others in the class.

## Report

Write a summary to the manager of the hospital, highlighting any problems that may occur and offer some solutions. Justify your summary by referring to the data the class has explored and your results.

# Further investigations

### Varying the server time

The server time is not necessarily fixed. Using your spreadsheets, explore different situations by randomising both arrival times and service times. For example, set up the number of people arriving to be between 0 and 5 each minute and the number of people served to be between 1 and 6. What happens?

### Traffic lights

Traffic lights are designed to limit the length of the queues that form at stop lights. Explore the systems used to improve traffic flow and avoid cars forming long queues at the lights.