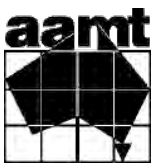
Australian Government
Department of Education, Science and Training

RESEARCH POINTERS TO PRACTICE

A review of research to inform middle years mathematics teaching and learning



Research Pointers into Practice

Text by Brian Doig

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The Australian Association of Mathematics Teachers Inc.
GPO Box 1729 Adelaide SA 5001
PHONE (08) 8363 0288
FAX (08) 8362 9288
EMAIL office@aamt.edu.au
INTERNET www.aamt.edu.au

CONTENTS

Foreword

This publication has been produced by the Australian Association of Mathematics Teachers to draw together important evidence from research that supports the teaching and learning of mathematics in the Middle Years of schooling. The AAMT thanks the Australian Government for supporting its development, through funding associated with the Quality Mathematics in the Middle Years National Conference in Perth in April 2005. Research Pointers to Practice will be a valuable resource to participants in that conference in the first instance and, subsequently, to teachers, schools and others involved in mathematics in the middle years.

The Beyond the Middle report signalled two priority areas for national initiatives in literacy and numeracy education for the middle years of schooling:

- the need for literacy and numeracy assessment practices to be more closely linked to teaching instruction and learning; and
- strengthening the role of assessment information in guiding curriculum development and reform.

In response to the Beyond the Middle report, the Australian Government has provided a total of $4.8 million for the Literacy and Numeracy in the Middle Years of Schooling Initiative. The focus of the Initiative is to increase teachers' knowledge, understanding and professional skill development about best practices in literacy and numeracy assessment, curriculum and teaching instruction for their particular middle years of schooling contexts. Under this Initiative, Australian Government funding is to be used to strengthen the role of literacy and numeracy assessment practices, in particular, strengthening the link between assessment practices, classroom based teaching and learning, and curriculum development.

The AAMT would like to acknowledge the generous assistance and advice offered by Dr Jill Vincent, Australian Government Numeracy Co-ordinator, and Dr Max Stephens from the University of Melbourne, during the writing of this work. Their generous contributions, from the forthcoming volume, *Numeracy research and development initiatives 2001 – 2004: An overview of the Numeracy projects* (Vincent, Stephens, & Steinle, Forthcoming), helped provide a sound basis for the present work.

The AAMT would also like to express its gratitude to Deirdre Schaeffer from the Department of Education, Science and Training, who provided invaluable and constructive feedback throughout the project.

'What are the practices of effective teachers of mathematics?' and 'What are the characteristics of effective mathematics teachers?' are questions that have become prominent in recent times, particularly with respect to the Middle Years. In the pages that follow, some of the research that provides directions to possible answers to these questions is described,

Research Pointers to Practice is intended to provide professional educators with an introduction to the findings of research that have the potential to impact favourably on student outcomes in numeracy during the Middle Years of schooling.

Why research pointers to effective practice?

Pointers from research for improving the effectiveness of classroom practices in numeracy are widely acknowledged as a key resource for educators. While Hill and his colleagues argue that the 'key to improved educational outcomes is teacher effectiveness' (1993, p. 30), accessing the wealth of research findings is often a difficult and complex task.

Research Pointers to Practice is an attempt to alter this situation.

Thus, the purpose of this book of *Research Pointers to Practice* is to

- ✓ raise issues in Middle Years mathematics teaching and learning
- ✓ describe the research that has suggested the pointers
- ✓ bring together as many of the relevant pointers as possible, and
- ✓ organize the pointers in a way that makes them useful to educational professionals.

In this way educators will be able to access, and use, those research pointers to enhance their effectiveness as teachers, and thus the effectiveness of students' learning.

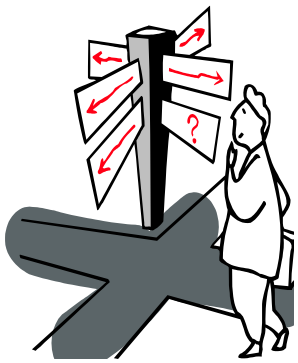
The book is divided into two complementary sections. The first section outlines and discusses several of the major issues concerning the teaching and learning of mathematics in the Middle Years.

The second section is the 'pointers' section. This is divided into sections that are for convenience only, as many pointers are applicable to more than one situation or issue. The 'pointers' sections are

- ✓ Pointers the Middle Years: Suggestions that focus on the primary to secondary transition phase
- ✓ Pointers from Abroad: Some ideas from overseas

- ✓ Pointers from Australia: A look at recent research funded by the Australian Government
- ✓ Pointers for Special Groups: Suggestions for students with particular needs
- ✓ Pointers to Teachers: Idea for teachers to try with their students
- ✓ Pointers to Actions: Ideas, that when put into action, give results
- ✓ Pointers to Mathematics: Ideas from research about specific mathematics topics
- ✓ Pointers to the Classroom: Suggestions that have worked in other classrooms
- ✓ Pointers to Broader Contexts: Ideas that come from research into wider professional contexts
- ✓ Where to from here?

Key sections of the pointers text are marked by ‘pointers’ like this, but in a smaller size.



In the pages that follow, the pointers to effective numeracy practice are drawn from recent research that examines ways of improving teaching and learning, not all of which is necessarily focused on the Middle Years *per se*, but from which broader lessons may be learned.

However, the research sources are restricted to those that come from International and Australian studies, carried out during the last decade, and recommendations not supported by research evidence are not included.

What is numeracy in Australia?

Numeracy is a key idea when considering school mathematics for the Middle Years, and how this is so, is revealed in the pages that follow.

Many people believe that there is a single, shared meaning for the term numeracy. However, the term is used in a number of different ways.

Numeracy, originally an English term, has little currency outside Britain and its former colonies, particularly Australia and New Zealand. Educators in other parts of the world use terms such as *school mathematic*s, *quantitative literacy,* or *mathematical literacy*, when wishing to speak of what we think of as numeracy.

In a move to improve communication, the states and Commonwealth have agreed upon a shared view of numeracy for Australian use. This derives from the Australian Government funded Numeracy Education Strategy Development Conference, conducted under the auspices of the Education Department of Western Australia and the Australian Association of Mathematics Teachers in 1997.

The published outcomes of the conference, *Numeracy = Everyone's Business*, state that

> numeracy involves … using … some mathematics … to achieve some purpose … in a particular context
>
> (Australian Association of Mathematics Teachers, 1997, p. 13)

In other words, numeracy is the effective use of mathematics

> to meet the general demands of life at home, in paid work, and for participation in community and civic life. Thus numeracy is:
>
> - distinct from literacy;
> - more than number sense;
> - not only school mathematics; and
> - cross-curricular.
>
> (Australian Association of Mathematics Teachers, 1997, p. 39)

Why a focus on the Middle Years?

The Middle Years of schooling, generally defined as Years 5 to 8, although in some cases this is extended to Years 4 to 9, has become a focus of attention for teachers, researchers, and school systems. Why is there this interest?

It comes as no surprise, to upper primary and junior secondary teachers, that for many students these middle years of schooling are an important transitional period in their lives that present special challenges and opportunities for teachers. It is now recognized that students in these years have special needs that are products of, or exacerbated by, changes in a student's social and emotional life.

Some of the needs of adolescent students were identified by Eyres (1992) as including the need for independence, developing self-identity and self-confidence.

At this time in their lives, when perhaps most vulnerable, students are also being bombarded by mixed messages from the media, parents, and peers about behaviour, while out of school experiences are becoming more relevant than those within school (Eyres, 1997). These needs raise issues for those involved in the education of Middle Years students.

There are growing concerns for the achievement of these students, particularly as the recent results of international studies in reading, mathematics, and science have indicated that Australian Middle Years students, despite continuing to achieve well, the performance of students from a number of other countries has improved, with the results that some of these have now overtaken Australia in the international standings. This demonstrates that there is considerable room for Australia to improve its performance. For example, the Australian results for the PISA and TIMSS studies, released recently, show that relative to students in some other countries, our Middle Years' students are not keeping up in some aspects of mathematics, a point made in each of the Australian TIMSS and PISA reports (see, Lokan, Ford, & Greenwood, 1996, 1997; Lokan, Greenwood, & Cresswell, 2001; Stacey, 1997, for comments on specific strengths and weaknesses of Australian Middle Years students).

In the Australian report on the TIMSS Video Study report, Hollingsworth and her colleagues suggest that in Australia there is a culture of 'shallow learning', and a lack of complex or higher order thinking (Hollingsworth, Lokan, & McCrae, 2003, p. xxi). Although similar issues have been raised in other countries (Felner, Shim, Favazza, & Seitsinger, 2000), this issue is not universal (Stigler & Hiebert, 1999) which begs the question about what are the pedagogical practices elsewhere, and also suggests that the current situation is neither inevitable nor incurable (see, Biggs, 1994, for suggestions about possibilities).

Research shows, too, that there is a growing gap between those students who see themselves as successful at school and those who do not (Lokan *et al.*, 2001). In part, this may be due to the fact that they are not succeeding

academically. The Cockcroft Report from the United Kingdom (Cockcroft, 1982), suggested that by age fourteen there is up to a 7-year difference in students' achievement in mathematics, while Hill and his colleagues noted that the lowest attaining students do not appear to progress academically beyond Year 4 (Hill *et al.*, 1993; Sullivan, 2003). Clearly, there are critical consequences of not taking these issues into account within the context of the mathematics classroom.

However, as the following pages demonstrate, these issues have received a considerable amount of attention from educational researchers, with the result that for every issue there are several pointers to effective solutions.

What are the Middle Years students' issues?

As is generally agreed by researchers, parents, and teachers, the life of a young adolescent — the Middle Years student — is one of transition and physical and emotional changes to previously established behaviours and activities. During this period of schooling there is the potential for more development of independent learning and higher order thinking, and perhaps an even greater need for relevance.

These changes are recognizable by their effect on students' learning and behaviour. In particular, at least three are major factors that will effectively prevent students from participating fully in the classroom.

Batten and Russell (1995) describe a trend towards increased rates of truancy in Australia, that effectively stops students from benefiting from anything that schools have to offer. Truancy is widely credited with the poor performance of Indigenous students, and there have been efforts to curb this truancy (see, for example, Callingham, 1999; and McCrae *et al.*, 2000).

However, truancy is not the only student strategy for avoiding school. Nardi and Steward describe their research in England into students who are quietly dis-affected and disengaged (2002a; 2002b) and how they may exist unnoticed. They use Oakley's (1999) term RHINO — Really Here In Name Only — to describe the quiet student who is disengaged but passes unnoticed in many busy classrooms. Importantly, Nardi and Steward maintain that their research underscores the fact that for this group of students, teachers can make a difference (see, also, Nardi & Steward, 2003).

These quietly disengaged students are the hidden members of a larger group of students who are alienated from school, but are more visible as they indulge in disruptive behaviours (Australian Curriculum Studies Association, 1996). Another aspect of this alienation and dis-engagement from school is that this problem appears more often in disadvantaged students (Lokan *et al.*, 2001). The cumulative effect of both disadvantage and dis-engagement can be a recipe for long-term problems for schooling and life in general.

Lokan and her colleagues also report (Lokan *et al.*, 2001) that academic performance in mathematics drops in the transition from primary to secondary school, regardless of the student's age when this transition occurs. This 'dip' occurs not only in mathematics, nor to Australian students only. However, as Biggs points out, this is not inevitable (Biggs, 1994).

Another, perhaps, less appreciated aspect of the issues for students moving through the Middle Years includes the on-set of puberty and all its attendant social and emotional upheaval. Arguably, it is not the responsibility of the mathematics teacher to guide students through this period, but attention to what psychological and emotional changes puberty brings, suggests that a more understanding approach to the needs of students is essential.

What are the Middle Years' mathematics issues?

Understanding that the Middle Years student has social and emotional needs is only one facet of Middle Years mathematics teaching and learning.

Improving students' achievements in Middle Years mathematics will only occur if we take into consideration aspects of students' engagement, the mathematics content, mathematics teaching and learning, and the important role played by the school community. Addressing each aspect is crucial in itself, and together they form a hurdle to successful mathematics achievement.

Belonging

A sense of belonging has been suggested as missing in those students whom we would define as dis-affected or disengaged from Middle Years mathematics.

However, as Boaler and her colleagues discovered (Boaler, Wiliam, & Zevenbergen, 2000), even successful students, in both US and UK secondary schools, reported that they disliked mathematics, as they did not see mathematics as being part of the person that they wanted to be. The authors suggest that it represented a 'life-path that is uninviting for most students. Students rejected mathematics as a thing to which they wished to belong.

This is, of course, somewhat different from the results reported by Siemon and her colleagues (Siemon, Virgona, & Corneille, 2001), where they found that success in mathematics was crucial to engagement, but not the only factor. The Australian research would appear to be far more positive in suggesting possible avenues for addressing this issue.

As Boaler *et al.* remind us (2000), to belong, in mathematics classes, involves being a member of a community of practice. This, in turn, requires that both teacher and student are engaged in dialogue (see, for example, Doig, 2003; Doig, Groves, & Splitter, 2000). Splitter and Sharpe (1995) provide many suggestions for creating better classroom dialogue, including that within the mathematics classroom.

Content

Both Luke *et al.* (2003) and Lokan *et al.* (2001) describe a 'dip' or 'plateau' in mathematics achievement in the junior secondary years, a finding similar to that from research in many Western countries. Possible causes may well be the emotional and social changes alluded to previously, but there may also be a mathematical aspect to this phenomenon. Students who are working confidently in whole numbers are suddenly expected to think relationally or algebraically: and many fail to surmount this hurdle.

In the reports on the achievements of Australian Middle Years students in international comparative studies, Lokan and her colleagues (Lokan *et al.,* 1996, 1997; Lokan *et al.,* 2001) have documented the apparent 'gaps' in achievement

of Australian students, and the difficulty students have with more complex mathematical procedures and more abstract mathematical ideas (see, Doig, 2001 for a summary of these findings).

Other researchers have re-inforced the findings that certain mathematical concepts and levels of thinking are common causes of difficulty for upper primary and junior secondary students. Siemon and her colleagues (2001), for example, found that students had most difficulty with vulgar and decimal fractions and multiplicative thinking. Similarly, Pearn and Stephens (2003) working with Middle Years students and vulgar fractions, reports the persistence of whole number concepts and thinking being applied to fractions. While these difficulties do exist, the possibility of improving students' conceptual thinking has been demonstrated Kazemi and Stipek (2001), who report that in their study of upper primary grades, teachers were able to develop sound conceptual thinking through classroom strategies.

This persistence of earlier learning is not surprising when one considers students' mathematical history, as the work of Hart and her colleagues in the UK attests (Hart, 2000, 1982; Johnson, 1989). Sowder (2000) reports for the US (and the story is very similar here) that students in the Middle Grades are asked to move from operating on whole numbers to operating on integers, to have a greater understanding of multiplication and division after years on addition and subtraction. Further, she claims that students are now expected to understand and use units in new and subtle ways: as composite units (such as four six-packs of drink), they create new types of unit quantities (such as lollies per child); and they are expected to use a fraction as a unit (such as a third of a pie is used as a unit (such as to allow two one-thirds to make two-thirds).

Clearly the problems identified are stumbling blocks to further development, and ways of overcoming them need to be explored.

How this may be accomplished has many answers: for example, based on an analysis of the US eighth grade results on TIMSS, Cogan and Schmidt (2003) recommended that the mathematics curriculum should be coherent across the grade levels, for all topics, and that fewer topics should be taught in the Middle School. Perhaps similar solutions may also be effective for solving the problems of Australian students, although not all researchers agree with Cogan and Schmidt's point of view (see Felner, Shim, Favazza, & Seitsinger, 2000, for example).

Learning

Closely connected to mathematics content issues are those issues dealing with learning. Several researchers have remarked upon the learning demanded of Middle Years students, particularly the level and quality of the intellectual demands on students (see, for example, Luke *et al.,* 2003).

The Australian reports of the OECD PISA programme (Lokan *et al.*, 2001; Thomson, Cresswell, & De Bortoli, 2004) for mathematics underscore the apparent lack of progress for many students during the Middle Years. Further, the Australian video-data from the TIMSS Video Study (Hollingsworth *et al.,* 2003) showed that about three-quarters of problems set for solution by students were deemed as being low in procedural complexity and repetitious. This was in marked contrast to the problems set for students in high achieving countries, where Japanese teachers, for example, used only 17% of this type of problem.

Hollingsworth and her colleagues suggested that there is a 'syndrome of shallow teaching' in Australia (p. xxi), the implication being a lack of complex or higher order thinking.

Teaching

The issue for teachers is how to address the multitude of Middle Years students' needs. There are many possible strategies for attacking this issue within the classroom, and the strategies below offer ideas about how this might be managed.

Clarke (2004) has suggested that asking higher order questions during lessons helps to make mathematics meaningful and challenging. He is not suggesting that these are mathematical questions for solution, but rather are the teacher's prompting questions. Typical prompts are: What happens if …?; What could you do next?; Is there quicker way?; Tell me about your pattern.

This type of question avoids putting a student 'on the spot' as it does not ask for a correct answer, but rather focuses on thinking, and strategies for solution instead. Promoting thinking is seen as a way to engage and challenge all students.

An extension of this idea is found in a further suggestion by Clarke to use rich assessment tasks. This type of assessment task is characterized, among other facets, by being engaging to students, connect to what has been taught, can be solved by more than one approach, and provide teachers with information about the specific needs of students.

The use of open-ended tasks, as suggested by Sullivan (May, 2003) has marked similarities to Clarke's suggestions above. In Sullivan's case, open tasks are those that focus students' attention on concepts, not merely procedures; allow students to decide, discuss, and select the strategies they wish to use; and can be used with students of varying levels of mathematical achievement.

Vale, working with a group of six primary and one secondary teacher, used an action research strategy to explore how to make Middle Years mathematics meaningful for students (Vale, 1999). Themes that arose from the strategies implemented were those of enjoyment and motivation; the engagement and belonging produced by the social interaction; and the fact that all students were able to experience success.

Schools

The role of the school community in the development of the mathematical achievement of Middle Years students is often taken for granted. The provision of basic resources and assistance, and so on, are seen as catering for students' needs without considering the issues in detail. This is very surprising when you consider how schools and school systems take great care to provide appropriately for the needs of children in the early years of schooling. Again, if as a society, we believe that the adolescent years are difficult, then how are schools taking appropriate actions?

Norton and Lewis (2000) produced a special report on Middle Grades reform in the Unites States for Phi Delta Kappa, in which they describe two successful school reform strategies.

In the first successful reform strategy, several facets were tracked. These included team planning (middle years teachers planning together); authentic teaching and learning (curriculum co-ordination, co-ordination of assessment, contact with parents and resource staff); and parent involvement (student achievement, homework social problems). Over the four years of the project, students in all schools improved academically in reading and mathematics, with students in the lowest SES schools improving most. Typically, the more co-ordination across grade levels, the higher the academic gains.

The second example of a reform endeavour is sited in a single school. In this school, teachers began with a series of 'self-study' professional development experiences, which have spurred a continuing programme of professional development, and the development of strategies to improve students' academic achievement. Other features of the school are the use of multi-age classes, and teachers staying with the same students as they move up grade levels. The success of this multi-faceted approach is evident in the achievements of the students in this school.

On a larger scale, Felner and his colleagues (2000) reported findings from a project on high performing learning communities. This research group employs a set of assessment instruments completed by teachers, students, and parents, known as the High Performance Learning Community (HiPlaces) Assessment. This suite of assessments gauges the degree to which a school implements the nine dimensions of High Performance Learning Communities (full-, partial-, or low-implementation). These nine dimensions include a high quality core curriculum that is rigorous, combined with high expectations of all students, and the development of small, personalized, learning communities, among others.

Students in the 31 project schools were also part of state-wide assessment programmes, and their results were compared to other, non-project schools. The state-wide tests are reported in a manner that allows comparison between schools. For mathematics, the state-wide mean was a scaled score of 250, and standard deviation of 50: high implementation schools had a mean score of 298, partial implementation schools a mean of 279, and the low implementation schools a mean of 248.

These results are, as the authors claim, impressive, and the strategy of the High Performance Learning Communities may well be worth further investigation.

In Victoria, the Middle Years Pedagogy Research and Development (MYPRAD) project used 'component mapping' to audit teachers' current practices. These components provide a picture of what effective Middle Years practice looks like, and guidelines for teachers in terms of strategies for moving towards more effective practices. The component mapping exercise involves individual teachers working through the components, with a significant other, to establish their level of practice with respect to each of the five components and their sub-components (25 elements altogether) (Department of Education and Training Victoria, 2004).

Tytler (2004) reported the analysis of the data from teachers in the MYPRAD project who were component mapped. The secondary teachers were from a

range of disciplines, whereas the primary teachers were generalist upper primary teachers.

From the mapping, the factors that teachers saw as needing more attention were listening to students, a more flexible curriculum, more links with the community, more variety in teaching strategies, and to move away from teacher-centred teaching.

It would appear that generally, a closer investigation of students' mathematical needs as the mathematical content changes is required (See, Stacey, 1997, for details and suggestions on this point). As students begin to study the concepts and procedures involved in vulgar and decimal fractions in the upper primary school, or commence the study of algebra in junior secondary school, better understanding of the subtleties of changes in content, the challenges of complex thinking, and the need for more personalized learning communities, are a challenge to everyone involved in Middle Years teaching and learning.

The Middle Years of schooling have become the focus of several research and development endeavours, both in Australia and internationally, in recent years.

Many of these research projects have come from the United States. For example, the *Connected Math Program* (CMP), in which curriculum materials have been developed in line with the National Council of Teachers of Mathematics standards for teaching, assessment, and curriculum (For details of these standards, see, National Council of Teachers of Mathematics, 1991, 1995, 2000).

Connected Math Program students improved their achievement scores on traditional tests of basic skills, and developed greater conceptual understanding and problem-solving skills (See, Doig, 2001, for details of the *Connected Math Program*).

Several key, relevant initiatives, however, have been conducted in Australia with support from the Australian Government.

Shaping Middle Schooling in Australia: A report of the National Middle Schooling Project (Barratt, 1997) was a response to the need for middle years-appropriate curriculum materials revealed by the *Alienation to Engagement Project* (Cumming, 1996). In her report Barratt stated that some schools had improved achievement for their middle years students, and supplied the guiding principles for effective middle years strategies. Barratt also cites the evidence-based findings of Lipsitz and Felner (1997) that reform that works

- ✓ has a single vision or big idea
- ✓ does not allow acceptance of casualties
- ✓ is long term and systematic
- ✓ is a co-ordinated effort
- ✓ changes culture and attitudes before changes are proposed
- ✓ is based on the ideas behind the changes, not the changes themselves.

(Adapted from Lipsitz & Felner, cited in Barratt, 1997, p. 41).

Similar findings are reported by Siemon from her research in Victoria. The *Middle Years Numeracy Research Project* (Siemon *et al.,* 2001), conducted during 1999 and 2000, pre- and post-assessed nearly seven thousand students from Year 5 to Year 9 in numeracy. An interesting finding from assessment data was that there was as much difference within schools as between schools, indicating that the opportunity to learn is as important a factor as student ability.

In the twelve-month interval between assessments, the twenty trial schools implemented an action plan that included a targetted programme focussing on numeracy. Eighteen of the twenty trial schools had a statistically significant improvement from use of their action plans and through the use of the project materials

The improvement in student achievement in the trial schools led the researchers to conclude that a difference can be made to numeracy outcomes, in the middle years particularly, when

- ✓ schools implement programmes that target numeracy
- ✓ teachers have shared beliefs and understandings about numeracy
- ✓ there is a whole school approach to planning for numeracy
- ✓ there is early diagnosis of difficulties and the implementation of intervention progammes.

In a comprehensive review of research and development projects in literacy and numeracy, Luke and his colleagues reviewed the relevant literature and analyzed the effectiveness of key literacy and numeracy teaching and learning strategies. In *Beyond the Middle: A report about Literacy and Numeracy development of target group students in the middle years of schooling* (Luke et al., 2003) Luke and his colleagues reported that effective schools tended to have

- ✓ dispensed with the traditionally accepted practice of placing the least experienced teachers in this level of secondary school
- ✓ strong school leaders with clear philosophies and a strong pedagogic and curricular focus
- ✓ integrated all students into the classroom, including those involved in withdrawal intervention programs

and that

- ✓ where interventions take the form of withdrawal programs, student gains are difficult to sustain unless the interventions are linked back into the mainstream classrooms.

The idea that some teachers may be more effective for their students than others has led researchers to look at student outcomes as a 'measure' of teacher effectiveness. The three studies described below provide good examples of this form of evidence-based research and its classroom usable outcomes.

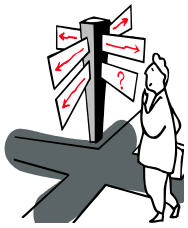
Research in England

A major study of effective primary mathematics teachers undertaken in England, the *Effective Teachers of Numeracy Study* (Askew, Brown, Rhodes, Johnson, & Wiliam, 1997) found that effective teachers had a set of coherent beliefs and understandings that underpinned their teaching.

Their beliefs related to '(a) what it means to be numerate, (b) the relationship between teaching and pupil's learning of numeracy, …[and] … (c) presentation and intervention strategies' (p 1).

Teachers in the study were interviewed about their teaching of mathematics and their style of interaction with students. The results of these interviews led to the definition of three models of teaching mathematics. These were *Connectionist*, *Transmission*, and *Discovery.* All but one of the highly effective teachers were classified as *Connectionist*, while teachers using other models were seen as only moderately effective.

Connectionist teachers based their practice on



- ✓ believing that almost all pupils are able to become numerate
- ✓ a belief that pupils develop by being challenged to think, explain, and solve problems
- ✓ valuing students' methods
- ✓ using students' understandings
- ✓ placing emphasis on making connections within mathematics
- ✓ developing rich connections between different mathematical ideas
- ✓ developing pupil's ability to select and use strategies that are efficient and effective.

The *Leverhulme Numeracy Research Programme* has confirmed some of the key results of the *Effective Teachers of Numeracy Study*. As Brown points out '[effective teaching] seems to be strongly related to the orientation of teachers, including beliefs and pedagogical content knowledge underpinning their practice, although not their formal subject qualification' (Brown, 2000, p. 6).

Research in the United States

In the study undertaken in the United States by Wenglinsky, using data from the United States National Assessment of Educational Progress (NAEP) (of grades four through twelve), the influences of several factors were examined. (2000). These factors included teacher professional development, classroom practices, student socio-economic factors, and class size. His significant findings included teacher practices and teacher characteristics.

For example, he found that those students who

- ✓ engaged frequently in hands-on learning outperform those who do not
- ✓ were exposed frequently to higher-order thinking skills outperform those who lack such exposure
- ✓ had teachers receiving professional development in higher-order thinking skills outperform students whose teachers had not
- ✓ performed at a higher level had teachers who had received professional development in working with different student populations
- ✓ achieved higher had a teacher who had major or minor university studies in the subject that they taught.

Wenglinsky's study also produced findings that were not significant, and these included number of years teaching experience, and whether the teacher had obtained a Masters degree or higher.

Cross-national studies

Cross-national studies can provide insights into the practices of teachers in many places and different circumstances. The largest such study to date is the Third International Mathematics and Science Study (TIMSS), conducted in 1995-1996. The TIMSS collected achievement data from students, and background information from school principals, teachers, and students. These data were linked so that the influences on the high-achieving students could be isolated and examined.

Martin and his colleagues (2000) reported on the findings of this investigation: these included that home factors (for example, parents' level of education) were the most significant difference between high- and low-achieving students. However, factors that relate directly to the student's school experience were also strong indicators of a student's likely success in the TIMSS achievement assessments.

Of the set of classroom factors that appeared to work consistently across countries, the most striking was that 'daily homework in a range of subjects (language, mathematics, and science) led to higher average achievement in science and mathematics' (2000, p. 11).

In their recommendations for reform of (United States) middle schools, based on these results, Cogan and Schmidt (2003) made five points

- ✓ That the curriculum should be coherent across the grades for all topics
- ✓ Fewer topics should be taught in the middle school
- ✓ Topics for study should be taught at an appropriate grade level
- ✓ Teachers should have clear and coherent standards of achievement to teach to
- ✓ The amount of review and repetition in lessons should be reduced.

Looking into classrooms

A further part of the Third International Mathematics and Science Study (TIMSS) was the TIMSS Video Study conducted by Stigler. This study observed Year 8 mathematics classrooms, randomly selected from within the TIMSS school sample, in three countries: Germany (100 classrooms), Japan (50 classrooms) and the United States (81 classrooms) (Stigler & Hiebert, 1997).

The analysis of the video-data enabled Stigler *et al* (1999) to claim that Japanese lessons, unlike those in the US, 'include high-level mathematics, a clear focus on thinking and problem solving, and an emphasis on students deriving alternative solution methods and explaining their solutions' (Stigler & Heibert, 1997, p. vii).

A similar study, the TIMSS 1999 Video Study, was conducted with seven countries, including Australia. In each school, one Year 8 mathematics and science lesson was video-taped (Hollingsworth et al., 2003). Eighty-three Australian secondary schools took part. While the video-data provide many insights into Australian teachers' practices in mathematics, there were no data collected to gauge the effectiveness of the participating teachers' practices. Hollingsworth and her colleagues suggest that 'students would benefit from more exposure to less repetitive, higher-level problems, more discussion of alternative solutions, and more opportunity to explain their thinking' (Hollingsworth et al., 2003, p. xxi).

Australian mathematics teachers are effective, as the overall performance of Australian students in international studies over the last forty years testifies, despite recent results where other countries appear to be catching up (Mullis, Martin, Gonzalez, & Chrostowski, 2004). (For a more complete description of Australian students' performances in TIMSS and other international and national studies, see, Doig, 2001).

Under the Australian Government's *Numeracy, a priority for all: Challenges for Australian schools* (Department of Education Training and Youth Affairs, 2000) several initiatives were funded under the *Numeracy Research and Development* Initiative to provide a basis for further action. While these initiatives were directed at the Early Years, they have implications for the Middle Years as well. These initiatives are divided into two categories: National Strand Projects and Strategic Strand (State and Territory) Projects.

National Strand Projects

The National Strand projects cover a wide range of perspectives and issues, ranging from numeracy development and effective practices in the pre-school years to a mapping of all relevant recent numeracy research in Australia.

The Numeracy in the early years: Project Good Start

This project examined what are effective early years practices in numeracy. *Project Good Start* found that, in more effective Child Care and Pre-school Centres, staff believed that numeracy skills need to be developed systematically, and had purposeful, structured numeracy programmes focused on counting, shape, size, length, weight, sorting, and sequencing. These programmes were also flexible to cater for the needs and interests of each child. *Project Good Start* noted that where staff were well-trained and had specific interests in early numeracy education, children were clearly enthusiastic about their numeracy skills.

Primary numeracy: A mapping and analysis of Australian research in numeracy at the primary school level

This project set out to map the extensive body of Australian primary numeracy research carried out during the last decade or so, and review this research in the international context. From this mapping, it was possible to build a comprehensive picture of current key research that could provide a basis for future planning and direction for further research.

Among the many findings of this review was that, despite the consistent positive findings in the research on various issues, little notice had been taken of these findings. For example, despite the positive evidence relating to children's development of number sense through the use of calculators, these are not being used as often as is possible, especially in the early years.

The *Mapping* project also reported that research on children attending schools in disadvantaged areas revealed a greater range in numeracy achievement than for children in other areas. The research examined by the review indicated also, that successful numeracy learning for children in these disadvantaged areas does occur when families and community members are involved in numeracy programmes and where

- ✓ teachers' expectations of children are high
- ✓ these high expectations were conveyed clearly to parents and other significant community members
- ✓ teaching is shaped by the results of teacher-child interviews
- ✓ open-ended learning tasks are used.

Home, school and community partnerships to support children's numeracy

This project gathered information on home, school and community partnerships that support children's numeracy in the two years prior to formal school, and in the primary years of schooling. The project focused on those partnerships that extended beyond schools and included other important contexts in which children live, develop, and learn.

The project reported that, despite the effort put into numeracy initiatives, more is needed in the form of community education campaigns to raise wider community awareness of numeracy, and emphasise the important role that families, and the general community, play in developing numerate citizens.

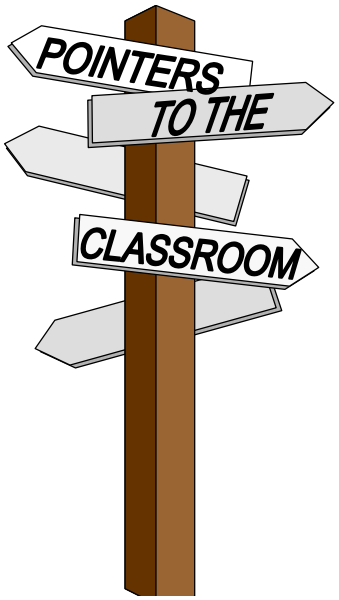
Numeracy: Families working it out together

This project sought to promote the importance of numeracy to parents by means of a poster and three brochures targetting different age groups — early years (including preschool) and the middle and later years of primary school. These efforts to raise awareness may take some time for their effects to have an impact in the general community.

Strategic Strand (State and Territory) Projects

The second category of projects, are as diverse in their perspectives on effective numeracy practices as are the *National Strand Projects.* These ten strategic projects have provided a wealth of interesting information about how schools, teachers, children, and the community view and apply numeracy.

These projects, and some of their more interesting findings, will be found in the appropriate sections of this review, and also in the Special Edition of AAMT's journal the *Australian Primary Mathematics Classroom* (Number 4, 2004).



The *Building mathematical understanding in the classroom: A constructivist approach* project (Department of Education Science and Training, DEST, 6) focused on the collection and analysis of rich case study data from three successful constructivist classrooms at Years 3, 4 and 5 and analysed those aspects of the classrooms which appeared to contribute to enhanced numeracy outcomes.

The researchers argue that the social and physical environment of the classroom play a major role in supporting students as they construct their understandings in numeracy. The social and physical settings of the effective mathematics classrooms studied created a welcoming climate, encouraged co-operative relationships and supported learning.

These observations are supported by the Victorian project (DEST, 7), which observed that where there was evidence of a mutually respectful learning environment in which both teacher and students were expected to share and justify their thinking, it was more likely that activities would be more deeply focused on the mathematics.

A key feature of the successful constructivist classrooms described in this project was that students were challenged in their thinking and understanding as the teacher supported them in building onto their prior knowledge.

The research reviewed suggests that effective teachers of mathematics establish and maintain classroom environments where students

- ✓ feel secure and safe
- ✓ are engaged in their own learning
- ✓ engage actively with ideas and evidence
- ✓ have high expectations of them
- ✓ are challenged to develop meaningful understandings
- ✓ are challenged to think, through explaining, listening, and problem solving
- ✓ have the opportunity to share their strategies
- ✓ have extended time to explore concepts
- ✓ have more focussed, sustained, and in-depth opportunities to learn
- ✓ are provided with appropriate feedback.

In considering what makes a mathematics teacher effective for their students, research reveals that, as one would expect, there is not a single 'type' of teacher but rather their are effective teachers with a many characteristics.

While it is highly unlikely that any single effective teacher possesses all of the characterristics, there are many that appear consistently in the research on effect.

The aims of the *What's making the difference in achieving outstanding primary school learning outcomes in Numeracy?* project (DEST, 1) were to determine the practices that are 'making the difference' in enabling primary school students to achieve outstanding outcomes in numeracy, and to assess the extent to which, and in what ways, these effective practices can be successfully applied in other school contexts.

This project revealed that the characteristics that contributed to enhanced numeracy outcomes for students were that effective teachers

- ✓ had high expectations of their students
- ✓ encouraged discussion during lessons to enable students to engage with and understand new and established mathematical concepts
- ✓ participated in collaborative planning with their colleagues
- ✓ worked where there was a whole school commitment to numeracy with all teachers implementing policies and programmes consistently in all classrooms.

The project report also noted also that effective teachers took mathematics lessons at a regular time each day, and used explicit teaching strategies for student engagement and learning.

Other research suggests that, in general, an effective teacher

- ✓ is fair in their dealings with students
- ✓ exhibits a caring attitude to students
- ✓ understands how their students learn
- ✓ values students' methods
- ✓ uses students' understandings
- ✓ understands the importance of the students' culture
- ✓ has a sense of humour
- ✓ is organised in their approach to teaching

- ✓ has a well-structured program with achievable goals
- ✓ has a sound knowledge of the mathematics
- ✓ models enthusiasm and interest in mathematics by example
- ✓ continually renews and reconstructs their goals and strategies.

Interestingly, the value of having high expectations of students is raised in many of the research studies. The importance of high expectations was highlighted by Bamburg (1994) in his monograph *Raising expectations to improve student learning.* Among other issues, he raises the issue of effort, as opposed to innate ability, by comparing Chinese, Japanese, and American attitudes to these two bases for learning. His evidence suggests that the Western notion of ability may well be the downfall of many students as 'their [learning] opportunities are substantially limited … when teachers have low expectations [of the students]' (p. 9).

In a recent review of effective practices in teaching mathematics in the pre-school and early years of schooling, Doig and his colleagues (2003) summarize in their *Sandpit Suggestions* that effective teachers of these children have high expectations of their students; plan lessons with a conceptual focus; have an understanding of their students' mathematical development; and provide the opportunity for students to share their solution strategies.

At a later stage of schooling, the Tasmanian project, *Improving Numeracy for Indigenous Students in Secondary Schools* (INISSS), had as its major objective the improvement of mathematical outcomes for all students, but particularly Indigenous students, in the middle years of schooling, through a program of intensive teacher professional development (Callingham, 1999 #120).

The professional development program was based around teachers' use of innovative tasks that posed realistic, intriguing and mathematically rich problems for students to solve. The results of this project showed that 'the program appears to have met its goal of improving numeracy outcomes for all students, but particularly those [outcomes] of Aboriginal students' (Callingham, 1999, p. 3).

The poor performance of Indigenous students is especially serious for students who live in remote communities, and whose first language is not English. Given the difficulties that these students experience with pencil-and-paper test formats, aside from inappropriate contextual and cultural issues, it is important that assessment tasks be developed for mathematics that will allow students to reveal what they know, and at the same time, give their teachers clear starting points for planning future educational experiences for the students.

For this reason, the *Supporting Indigenous students' achievement in numeracy* project (DEST, 9, forthcoming) explored these issues with students in several remote Indigenous communities. The project aimed to develop a bank of assessment tasks suitable for Indigenous students in the middle years, in remote non-urban schools.

A bank of appropriate effective assessment tasks, for teachers to use, together with protocols for evaluating students' performance encourages a whole school approach for using assessment information to plan for more effective teaching in these remote schools.

The survey of non-capital *Strategic Results Projects* of the *Indigenous Education Strategic Initiatives Programme* has revealed a wide range of teaching practices that has led to a wealth of achievement by Indigenous students. In *What works? Explorations in improving outcomes for Indigenous*

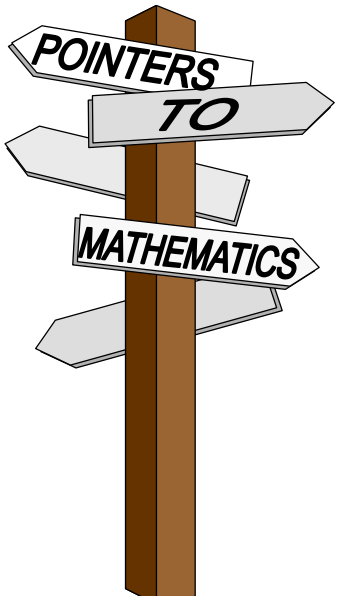
students (M^{c}Crae *et al.,* 2000) the extent of the *Strategic Results Projects* (SRP) aimed at improving the educational opportunities for Indigenous students is set out in great detail. These projects, in pre-schools to the VET sector, related to literacy, mathematics, vocational education, as well as other areas of education. These projects show what can be achieved in 'a relatively short space of time through concerted efforts' (DETYA, 1997 cited in M^{c}Crae *et al,* 2000, p. 12).

Typical of the results reported from the SRPs are the results at one site where the target was to achieve 80% of students moving one or more levels on the national Mathematics Profile in an academic year. Eight of 11 students (72%) did so. (M^{c}Crae *et al*, 2000, p. 93). where each level represents approximately 18 months normal school progress (p 153).

While the M^{c}Crae *et al* report, and the results of the Tasmanian project, both indicate that targetted actions by teachers can have positive results Malin (2000) suggests that the teachers who will be most effective will be those who 'have high expectations of Aboriginal and Torres Strait Islander students, who understands their students well, and who see themselves as learners, also open to new understandings from both their students and the parents of their students' (p. 351).

In their report on children with learning difficulties, Louden *et al.* (2000) suggest key, general strategies that they believe would improve the achievements of all students. These strategies are that

- ✓ there be a commitment to quality mathematics teaching in the early years
- ✓ more attention be paid to identification of children who encounter learning difficulties in mathematics
- ✓ there be a commitment to early intervention programs
- ✓ there be support for children who continue to encounter difficulties in the later years of schooling.



Concrete materials, such as those used in the Base Ten Game, have long been used to develop students' conceptual and procedural knowledge about the decimal number system. The report of the *Understanding place value: A case study of the Base 10 game* project (DEST, 7), however, suggests that a single type of concrete material, or one learning task, highlights only certain aspects of the number system, while a deep understanding requires that students experience a range of concrete materials and learning tasks. The report also suggests that teachers must help students to make the connection between concrete materials and learning tasks, and mathematical concepts.

The project researchers also asserted that teachers were better able to diagnose students' misconceptions and adapt learning tasks, to meet the learning needs of students, when they had developed their own understanding of the number system to a relational level.

The *Developing computation* project (DEST, 8) investigated the use of informal paper-and-pencil methods that extend mental computation methods, and precede the development of formal written algorithms. The project aimed to support Years 2 to 4 students' development of informal written methods, of computation, and see what effects this had on students' number sense and computational ability. Supporting this, the project wished to determine which classroom strategies that encourage informal written computation, are most effective in developing students' number sense and computational ability.

The approach to computation, in this project, was to

- ✓ emphasize children's informal strategies for mental computation
- ✓ have children share, orally, explanations of their strategies
- ✓ build on known mental strategies, by extending these using pencil and paper, for larger numbers.

The *Developing computation* project teachers indicated that the focus on mental computation, rather than the use of formal written algorithms, allowed students to develop a better understanding of numbers and place value, and provide a much more secure basis for a later introduction of the written algorithms.

The researchers also report that all the project teachers agreed that

- ✓ the concentration on mental computation had greatly increased children's competence and confidence in handling numbers and in understanding place value
- ✓ the development of informal written methods, as a bridge between mental and formal written methods, helped children to develop ways of dealing with larger numbers
- ✓ seeing their own thinking strategies in writing clarified students mental strategies
- ✓ the use of written work helped teachers to 'see' children's thinking and to work on mis-understandings.

The teaching and learning framework *Productive Pedagogy* (Gore, 2002; Queensland School Restructuring Longitudinal Study, 2001) is a generalized framework that is tightly focussed upon the teacher, and teacher's actions.

The four facets of Productive Pedagogy (Gore, 2002, p. 13) are intellectual quality, relevance, a supportive classroom environment, and recognition of difference.

Many of the details of the four *Productive Pedagogy* facets summarize the findings of research into the classroom actions of effective mathematics teachers.

While these actions are many and varied, the research indicates that the actions listed below are taken more often by effective teachers of mathematics.

That is, effective teachers more often

- ✓ reflect on their own practice
- ✓ challenge their students
- ✓ make learning experiences interesting and enjoyable
- ✓ engage students in making sense of they are learn
- ✓ focus on meaning and understanding
- ✓ encourage student persistence in learning
- ✓ implement sequences of lessons rather than single one-off activities
- ✓ teach for higher-level cognitive processes and strategies
- ✓ support students in need of additional assistance
- ✓ make explicit connections between ideas
- ✓ provide opportunities for practice
- ✓ continually audit their students' needs
- ✓ believe that assessment is a key facet of teaching and learning
- ✓ employ fair assessment and reporting processes
- ✓ embed assessment within the learning strategy
- ✓ use assessment tasks that are directly related to classroom tasks
- ✓ provide feedback that is timely and appropriate.

The three main purposes of the *Researching Numeracy Teaching: Approaches in Primary Schools* project (DEST, 2) were to

- identify the key components of teaching approaches that lead to improved mathematics outcomes for students in the primary years
- to determine if implementation of a defined suite of numeracy teaching approaches results in improved learning outcomes
- to determine how these successful approaches can best be presented to teachers so that they may implement them in their own classrooms to improve student learning.

In analyzing effective teaching practices, the research team defined the term 'teaching approach' as 'an orientation to teaching that plans to make use of particular scaffolding practices to achieve a specific purpose related to mathematics learning'. Throughout the life of the project the research team made numerous observations during visits to schools as they visited and worked with teachers from several schools over nearly two years.

These observations, and their analysis, led the researchers to assert in the project report that

- ✓ where groups of teachers (or indeed the entire school staff) had made a significant effort to plan together on a regular basis, teachers reported more positively on shifts in their practice and confidence, as well as improvements in student mathematics learning
- ✓ strong leadership appears needed to elevate individual practice to shared practice and to make good practice an object of inquiry and examination
- ✓ a major outcome of the project was the identification, description and elaboration of twelve scaffolding practices that appear to be effective in improving student learning outcomes.

In terms of assessment actions that could improve teaching and learning, the *Assessing numeracy in primary schools* project (DEST, 10) examined how teachers could make the best use of large-scale assessment programmes, and classroom assessments. The project looked at ways in which teachers can

- ✓ use the results from system and school assessments to improve their classroom practice
- ✓ identify classroom assessment practices that complement system and school assessment
- ✓ develop teaching approaches that make effective use of assessment to support learning.

In addition to the Pointers detailed above, there are contextual factors to effective practice in mathematics that characterize the effective school and its staff.

Research has shown that these factors include relationships between colleagues and the school community, and the quality of the school leadership.

Professional relationships

Those teachers who are deemed to be effective have at least some of the following characteristics in professional dealings with colleagues.

They endeavour to

- ✓ share clear and coherent standards of achievement
- ✓ share their norms and values
- ✓ collaborate with their colleagues
- ✓ support their colleagues
- ✓ engage in reflective dialogue
- ✓ have the support of the broader school community
- ✓ engage in professional development activities.

School leadership

Effective schools in mathematics have school leadership that encourages and supports effective teaching and learning through

- ✓ employing a whole-school approach to school policies and practices
- ✓ providing a coherent program throughout the school
- ✓ providing time for staff to plan
- ✓ providing a positive working environment for staff
- ✓ encouraging staff to undertake professional development
- ✓ communicating school policies and practices to the wider community.

There are some factors, that have shown consistently, that impact on the improvement of numeracy results in Year 3 to Year 5 for individual students, and the *Profiling High Numeracy Achievement* project (DEST, 5) sought to identify the teaching practices and school programmes that supported these.

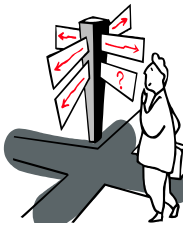
Based on the findings of this, and other research, the researchers conducting this project claim that substantial and enduring changes to numeracy practices require a whole school approach to ensure an integrated and coherent programme, and teachers need access to both professional development and appropriate resources.

Professional and collegial issues

The *Teachers enhancing numeracy* project (DEST, 3) aimed to determine the elements of numeracy learning environments that enhance student numeracy outcomes; to investigate the effects of collaborations between academics and teachers on numeracy teaching practices and student outcomes; and to identify the elements of professional development that enhance student numeracy outcomes. These factors were explored using children's numeracy outcomes as measured by pre- and post-tests.

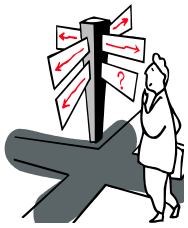
Knowing the elements of professional development that enhance student numeracy outcomes was identified by the project as a key factor in improving teaching practices. The key elements of professional development identified by the project were:

- ✓ expert input
- ✓ 'just-in-time' support from peers
- ✓ peer sharing
- ✓ classroom-based focus.



The *Numeracy across the curriculum* project (DEST, 4) had as one of its aims to identify how teachers can assist students cope with numeracy demands, and in improving their numeracy levels.

The project findings suggest that there needs to be time for new ideas to become incorporated into teachers' thinking, for ideas to be tried in action in teachers' own classrooms, and time for continuing professional reflection and collegial discussion.



Further, the researchers suggest that more attention needs to be given to the development of appropriate assessments for measuring students' achievement in numeracy across the curriculum.

Where to from here?

While it is tempting to claim that the research evidence is clear and unequivocal, the reality is that teaching and learning are such complex processes that the best that can be offered is the result of careful observation of what effective teachers do, even though what they do can be very different over time and place.

The wide range of research reviewed in this book points to possible solutions, both for the Middle Years, and other stages of schooling, for many of the major issues confronting educators in the twenty-first century. Some of the major questions that confront us are presented below, in a slightly provocative manner. What is your position with respect to these?

Assessment

In the context of effective teaching and learning, assessment is seen by many as the critical influence. The work of Black and Wiliam, for example, provides insights, suggestions and challenges, with respect to re-siting assessment centrally to the educative process (Black, 1999; Black, Harrison, Lee, Marshall, & Wiliam, 2002; Black & Wiliam, 1998; Wiliam, 2001). The Australian researchers Clarke and Sullivan contribute possibilities for effective educational experiences (Clarke, 2004; Sullivan, 2003). But are we all convinced that assessment should drive our practice as some would have it (Barnes, Clarke, & Stephens, 2000)?

Adolescent development

This issue, and the likely related issue of dis-engagement, can be addressed, it has been suggested, by making the mathematics more relevant to students' interests, a suggestion that has a common sense ring to it. The experience of schools in the Victorian Middle Years Pedagogy Research and Development Project (Tytler, 2004) would suggest that student interests are a powerful force for student engagement. The question is, how to harness this force?

Curriculum

Clearly, the content of the mathematics curriculum can effect engagement, link to student interests, be relevant, and inclusive. While not every aspect may be addressed equally, curriculum can be made to work for, not against the teacher and the learner. While examples of effective curriculum are to be found in schools around the country, and subject associations, like AAMT, provide spaces where these examples can find a voice, perhaps wider access to these effective practices is needed to help us all be more effective?

Classroom issues

The classroom pointers from research are many and varied, however, the issue of 'what works for me' may not work for you, is a constant problem in sharing our good practices. Perhaps we need to have more ways of sharing our

practice, ones that allow for the variation in the contextual factors that may be affordances or constraints in someone else's classroom? Generic solutions may be more problematic, and less effective, than their proponents will admit.

Inclusion and diversity

That rural, remote, low SES, and Indigenous students, and other excluded and disadvantaged groups of students fail to benefit fully from schooling is unquestioned. Research and reports from surveys have pin-pointed both the issues and some possible solutions with respect to these students. Major investigations of students with learning difficulties, such as that conducted by Louden and his colleagues (Louden *et al.,* 2000), suggest that there are strategies that work for some of these students. Likewise, for those students with disadvantages of income, geography or race, research has identified strategies that work for some students: are these are worth further investigation?

Teacher issues

The issue for teachers is how to find those research programmes that have demonstrated the effectiveness of particular classroom strategies in Middle Years mathematics. That there are programmes that focus on effective teaching, for example, of working with decimal fractions (Steinle, Stacey, & Chambers, 2002), or the use of open-ended tasks (Sullivan & Lilburn, 2004), is not in question. The question is how do we find out about these strategies?

While many of these programmes have been summarized by Doig (2001), others were not. The *Primary numeracy: A mapping and analysis of Australian research in numeracy at the primary school level* (Groves *et al*., Forthcoming) project should be a key resource for addressing this issue.

Should there be a national clearinghouse of ideas, projects, or strategies to facilitate dissemination of relevant work?

The *Teachers enhancing numeracy* project (DEST, 3) found, among other factors, that students benefitted when their teachers had deep mathematical content and pedagogical knowledge, and maintained effective classroom inquiry approaches to learning. However, research suggests also that these factors need to be complemented by an understanding of the place of mathematics in human history, and its contribution the development of our modern society.

School organization

In the United States teachers are not the only focus of reform in the Middle Years: whole schools are. The work of researchers such as Felner (Felner *et al.,* 1997; 2000) suggest that coherent whole school approaches are more effective than other reform strategies. This has also been shown true for other year levels: for example in the work of Slavin (Slavin, 1999). The suggestion is, therefore, can there be Middle Years mathematics reform without the support of the school administration?

All of these issues and questions invite exploration by all concerned with the mathematical education of students: however, the reality is that it is the individual teacher in the classroom who will, in the end, make the difference.

Resources

The listings below are a small selection of useful sources for ideas for Middle Years teachers.

- ✓ Education Network Australia (EdNA) at http://www.edna.edu.au/edna/go/pid/1
- ✓ The Australian Association of Mathematics Teachers (AAMT) at http://www.aamt.edu.au
- ✓ All State and Territory education web-sites
- ✓ National mathematical association web-sites in the United Kingdom and the United States
- ✓ *The Australian Mathematics Teacher.* Australian Association of Mathematics Teachers Inc. Adelaide (Australia): AAMT Inc.
- ✓ Australian Association of Mathematics Teachers Inc. (2005). *Making Mathematics Vital: Proceedings of the 20th Biennial Conference of the Australian Association of Mathematics Teachers Inc.* Adelaide (Australia): AAMT Inc.
- ✓ The Australian Journal of Middle Schooling
- ✓ Clarke, D. (1996). *The Case of the Mystery Bone*. North Ryde, NSW (Australia): The Mathematical Association of New South Wales Inc.
- ✓ Flewelling, G. with Higginson, W. (2005). *Teaching with Rich Learning Tasks: A Handbook (2nd edition)*. Adelaide (Australia): Australian Association of Mathematics Teachers Inc.
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- ✓ Parke, C.S., Lane, S., Silver, E.A. and Magone, M.E. (2003). *Using Assessment to Improve Middle-Grades Mathematics Teaching and Learning*. Reston, VA (USA): National Council of Teachers of Mathematics.
- ✓ Steinle, V., Stacey, K. and Chambers, D. (2002). *Teaching and Learning about Decimals (CD-ROM)*. Melbourne (Australia): The University of Melbourne.
- ✓ Sullivan, P. and Lilburn, P. (2004). *Open-ended Maths Activities: Using 'good' questions to enhance learning in Mathematics (2nd ed.).* Melbourne (Australia): Oxford University Press.

And, for those who are interested, the references overleaf provide a good start for further enquiry.

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